Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*1720169545*

**FURTHER MATHEMATICS** **9231/21**

Paper 2 **May/June 2017**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **24** printed pages and **4** blank pages.

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CAMBRIDGE International Examinations

**[Turn over**

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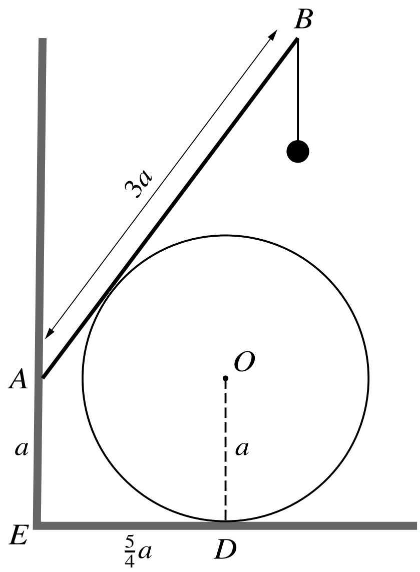
**1** A bullet of mass 0.08 kg is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed 300 m $s^{-1}$ and emerges horizontally after 0.02 s. There is a constant horizontal resisting force of magnitude 1000 N. Find the speed with which the bullet emerges from the barrier. [3]

**2**




A uniform smooth disc with centre $O$ and radius $a$ is fixed at the point $D$ on a horizontal surface. A uniform rod of length $3a$ and weight $W$ rests on the disc with its end $A$ in contact with a rough vertical wall. The rod and the disc lie in a vertical plane that is perpendicular to the wall. The wall meets the horizontal surface at the point $E$ such that $AE = a$ and $ED = \frac{5}{4}a$. A particle of weight $kW$ is hung from the rod at $B$ (see diagram). The coefficient of friction between the rod and the wall is $\frac{1}{8}$ and the system is in limiting equilibrium. Find the value of $k$. [8]

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3 Two uniform small smooth spheres $A$ and $B$ have equal radii and masses $3m$ and $m$ respectively. Sphere $A$ is moving with speed $u$ on a smooth horizontal surface when it collides directly with sphere $B$ which is at rest. The coefficient of restitution between the spheres is $e$.

**(i)** Find, in terms of $u$ and $e$, expressions for the velocities of $A$ and $B$ after the collision. [3]

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Sphere $B$ continues to move until it strikes a fixed smooth vertical barrier which is perpendicular to the direction of motion of $B$. The coefficient of restitution between $B$ and the barrier is $\frac{3}{4}$. When the spheres subsequently collide, $A$ is brought to rest.

**(ii)** Find the value of $e$. [7]

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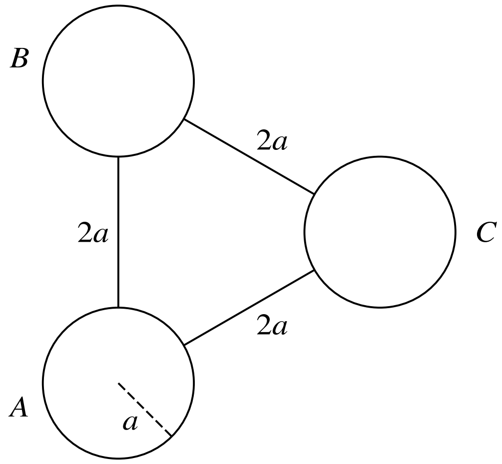


Three identical uniform discs, $A$, $B$ and $C$, each have mass $m$ and radius $a$. They are joined together by uniform rods, each of which has mass $\frac{1}{3}m$ and length $2a$. The discs lie in the same plane and their centres form the vertices of an equilateral triangle of side $4a$. Each rod has one end rigidly attached to the circumference of a disc and the other end rigidly attached to the circumference of an adjacent disc, so that the rod lies along the line joining the centres of the two discs (see diagram).

**(i)** Find the moment of inertia of this object about an axis $l$, which is perpendicular to the plane of the object and through the centre of disc $A$. [6]

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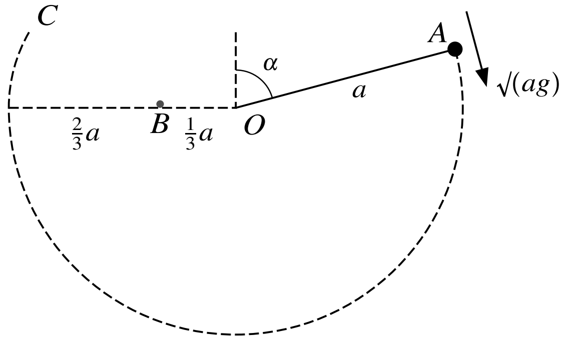
The object is free to rotate about the horizontal axis $l$. It is released from rest in the position shown, with the centre of disc $B$ vertically above the centre of disc $A$.

**(ii)** Write down the change in the vertical position of the centre of mass of the object when the centre of disc $B$ is vertically below the centre of disc $A$. Hence find the angular velocity of the object when the centre of disc $B$ is vertically below the centre of disc $A$. [4]

5




A particle of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The point $A$ is such that $OA = a$ and $OA$ makes an angle $\alpha$ with the upward vertical through $O$. The particle is held at $A$ and then projected downwards with speed $\sqrt{(ag)}$ so that it begins to move in a vertical circle with centre $O$. There is a small smooth peg at the point $B$ which is at the same horizontal level as $O$ and at a distance $\frac{1}{3}a$ from $O$ on the opposite side of $O$ to $A$ (see diagram).

**(i)** Show that, when the string first makes contact with the peg, the speed of the particle is $\sqrt{\left(ag(1 + 2\cos\alpha)\right)}$. [2]

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The particle now begins to move in a vertical circle with centre $B$. When the particle is at the point $C$ where angle $CBO = 150°$, the tension in the string is the same as it was when the particle was at the point $A$.

**(ii)** Find the value of $\cos\alpha$. [10]

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**6** A fair die is thrown repeatedly until a 6 is obtained.

**(i)** Find the probability that obtaining a 6 takes no more than four throws. [2]

**(ii)** Find the least integer $N$ such that the probability of obtaining a 6 before the $N$th throw is more than 0.95. [3]

7 A farmer grows a particular type of fruit tree. On average, the mass of fruit produced per tree has been 6.2 kg. He has developed a new kind of soil and claims that the mean mass of fruit produced per tree when growing in this new soil has increased. A random sample of 10 trees grown in the new soil is chosen. The masses, $x$ kg, of fruit produced are summarised as follows.

$$\Sigma x = 72.0 \qquad \Sigma x^2 = 542.0$$

Test at the 5% significance level whether the farmer's claim is justified, assuming a normal distribution. [7]

**8** The continuous random variable $X$ has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{4}(x-1) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

**(i)** Find the distribution function of $X$. [3]

The random variable $Y$ is defined by $Y = (X-1)^3$.

**(ii)** Find the probability density function of $Y$. [4]

**(iii)** Find the median value of $Y$. [3]

**9** Two fish farmers $X$ and $Y$ produce a particular type of fish. Farmer $X$ chooses a random sample of 8 of his fish and records the masses, $x$ kg, as follows.

1.2  1.4  0.8  2.1  1.8  2.6  1.5  2.0

Farmer $Y$ chooses a random sample of 10 of his fish and summarises the masses, $y$ kg, as follows.

$$\Sigma y = 20.2 \qquad \Sigma y^2 = 44.6$$

You should assume that both distributions are normal with equal variances. Test at the 10% significance level whether the mean mass of fish produced by farmer $X$ differs from the mean mass of fish produced by farmer $Y$. [10]

**10** A random sample of 5 pairs of values $(x, y)$ is given in the following table.

| $x$ | 1 | 2 | 4 | 5 | 8 |
|---|---|---|---|---|---|
| $y$ | 7 | 5 | 8 | 6 | 4 |

**(i)** Find, showing all necessary working, the equation of the regression line of $y$ on $x$. [4]

**(ii)** Find, showing all necessary working, the value of the product moment correlation coefficient for this sample. [3]

**(iii)** Test, at the 10% significance level, whether there is evidence of non-zero correlation between the variables. [4]





**[Turn over**

**11** Answer only **one** of the following two alternatives.

**EITHER**

A particle $P$ of mass $3m$ is attached to one end of a light elastic spring of natural length $a$ and modulus of elasticity $kmg$. The other end of the spring is attached to a fixed point $O$ on a smooth plane that is inclined to the horizontal at an angle $\alpha$, where $\sin\alpha = \frac{2}{3}$. The system rests in equilibrium with $P$ on the plane at the point $E$. The length of the spring in this position is $\frac{5}{4}a$.

**(i)** Find the value of $k$. [3]

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The particle $P$ is now replaced by a particle $Q$ of mass $2m$ and $Q$ is released from rest at the point $E$.

**(ii)** Show that, in the resulting motion, $Q$ performs simple harmonic motion. State the centre and the period of the motion. [6]

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**(iii)** Find the least tension in the spring and the maximum acceleration of $Q$ during the motion. [5]

 

**OR**

A shop is supplied with large quantities of plant pots in packs of six. These pots can be damaged easily if they are not packed carefully. The manager of the shop is a statistician and he believes that the number of damaged pots in a pack of six has a binomial distribution. He chooses a random sample of 250 packs and records the numbers of damaged pots per pack. His results are shown in the following table.

| Number of damaged pots per pack ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 48 | 69 | 78 | 32 | 22 | 1 | 0 |

**(i)** Show that the mean number of damaged pots per pack in this sample is 1.656. [1]

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The following table shows some of the expected frequencies, correct to 2 decimal places, using an appropriate binomial distribution.

| Number of damaged pots per pack ($x$) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Expected frequency | 36.01 | 82.36 | $a$ | 39.89 | $b$ | 1.74 | 0.11 |

**(ii)** Find the values of $a$ and $b$, correct to 2 decimal places [5]

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(iii) Use a goodness-of-fit test at the 1% significance level to determine whether the manager's belief is justified. [8]

 

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